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CS4423 Assignment II

Q1. A matrix is called "singular" if it doesn't have an inverse.
If a matrix is invertible, then all its eigenvalues are non-zero.

$$Av = \lambda v$$

All the rows of any laplacian matrix will sum to $\emptyset$ as each row contains the degree of a node and a -1 for each of its neighbours, same for columns as its symmetric. $\Rightarrow$ $L \cdot \mathbf{1} = \emptyset$, where $\mathbf{1}$ is the vector of all 1s.
This gives us an eigenvector $(1, 1, 1, \dots)$ for L and corresponding eigenvalue $\emptyset$, hence L is not invertible.

Q2 Perron-Frobenius theorem applies to square, non-negative, irreducible matrices.
Laplacian matrices contain -1s and are therefore not non-negative.

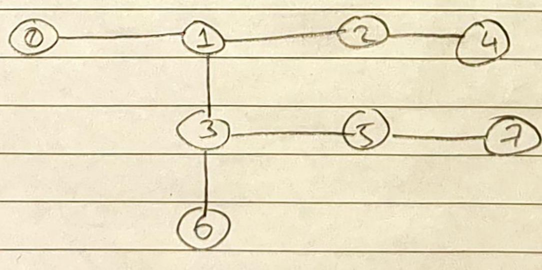
Q3 A matrix with all non-negative eigenvalues is called positive semi-definite; i.e., the real number $x^T M x$ is positive for every non-zero real column vector x.

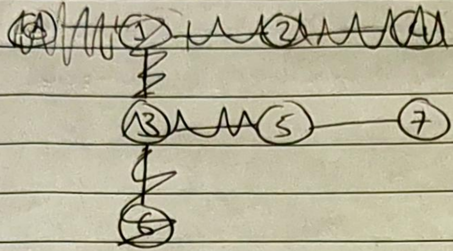
$$x^T L x = \sum_{\{i,j\} \in E} (x_i - x_j)^2$$

Essentially summing the squares of the differences over all edges.
$\hookrightarrow$ Always non-negative. $\Rightarrow$ $x^T L x \geq \emptyset$.

Q4. P_5 =

Q5

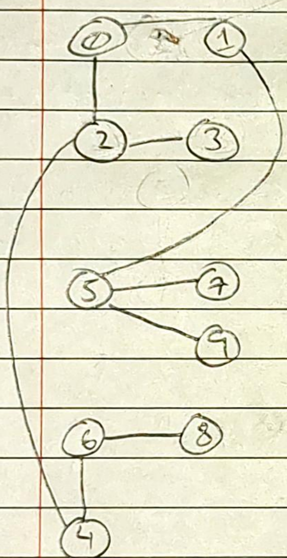


Q6 

$a = 121335$

Q7. $a = 2, 5, 6, 4, 2, 0, 1, 5$

$$
\begin{array}{ll}
 & 0123456789 \\
d = & 2231232111 \\
d_1 = & 2220232111 \\
d_2 = & 2220222011 \\
d_3 = & 2220221001 \\
d_4 = & 2220120001 \\
d_5 = & 2210020001 \\
d_6 = & 1200020001 \\
d_7 = & 0100020001 \\
d_8 = & 0000010001
\end{array}
$$



Q8 0, 1, 5, 7, 9, 2, 3, 4, 6, 8

Q9 0, 1, 2, 5, 3, 4, 7, 9, 6, 8