NUI Galway
OÉ Gaillimh

Summer Examinations 2016/17

Exam Code(s)	4BS2, 4FM2, 4BCT1, 4BMS2, 3BME1, 4BME1
Exam(s)	4th Science
Module(s)	Networks
Module Code(s)	CS423
External Examiner(s)	Prof. M. Lawson
Internal Examiner(s)	Prof. G. Ellis *Prof. G. Pfeiffer
Instructions	Answer *all* questions.
Duration	2 hours
No. of Pages	4 pages (including this cover page)
Discipline	Mathematics

Requirements:

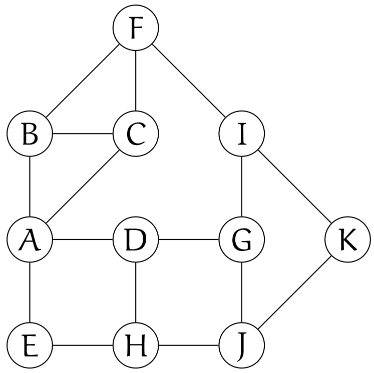
Release to Library	Yes ☑	No ☐
Release in Exam Venue	Yes ☑	No ☐
MCQ	Yes ☐	No ☑
Statistical/Log Tables	Yes ☑	No ☐

1.

(a) [**10** marks]

(i) Describe Breadth First Search as an algorithm for undirected graphs. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?

(ii) In the network below, apply Breadth First Search to determine a spanning tree with root G and the shortest distances from node G to each of the other nodes in the graph.



(b) [**10** marks]

(i) An edge AB in a graph is called a *bridge* if deleting this edge would cause the nodes A and B to lie in different connected components of the graph. What, in contrast, is a *local bridge*?

(ii) Suppose that a graph has two types of edges: strong ties and weak ties. A node A of the graph is said to satisfy the *Strong Triadic Closure Property* if, for any two nodes B and C, there is an edge (weak or strong) between B and C whenever A has strong ties with both A and B.

Show that, if a node A in the graph satisfies the Strong Triadic Closure Property and is involved in at least two strong ties, then any local bridge that A is involved in is necessarily a weak tie.

2.

(a) [**10** marks]

(i) Provide a formal definition of a finite n-player game.

(ii) What is a *best response* and what is a *Nash equilibrium* in a game?

(iii) For the following 2-player game, find all pure strategy Nash equilibria. The rows here correspond to player A's strategies, and the columns to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

	L	M	R
U	1, 1	2, 3	1, 6
M	3, 4	5, 5	2, 2
D	1, 10	4, 7	0, 4

(b) [**10** marks]

(i) Describe a sealed-bid second price auction as a game.

(ii) What is a dominating strategy for a bidder in a sealed-bid second price auction? Justify your answer.

(iii) A seller wishes to sell an item, which she values at s, in a sealed-bid second price auction. There are two buyers, who value the item at v_1 and v_2, respectively. Both buyers know that the seller will submit her own sealed bid of s, but they do not know the value of s. What is an optimal strategy for a bidder in this auction? Explain your answer.

3.

(a) [**10** marks]

(i) What is a bipartite graph by definition? And how can bipartite graphs be characterized in terms of cycles?

(ii) What does the Matching Theorem say about a bipartite graph?

(iii) Sketch a proof of the Matching Theorem.

(b) [**10** marks]

(i) Describe the concept of market clearing prices between a set of sellers and a set of buyers.

(ii) Suppose there are three sellers, A, B and C, and three buyers, X, Y and Z. Each seller is offering a house for sale, and the valuations of the buyers for the houses are as listed in the following matrix.

	A	B	C
X	3	6	4
Y	2	8	1
Z	1	2	3

Describe briefly the bipartite graph auction procedure. Apply this procedure to the above valuations in order to obtain a set of market clearing prices.

PTO.

4.

(a) [**10** marks] Suppose that $\mathsf{G} = (\mathsf{X}, \mathsf{E})$ is a directed network with node set X and edges $\mathsf{E} \subseteq \mathsf{X}^2$.

(i) Define what it means for G to be *weakly connected*, and what it means to be *strongly connected*.

(ii) In general, a directed network is partitioned into strongly connected components. Describe the equivalence relation that yields strongly connected components as its equivalence classes in terms of E, regarded as a relation on X.

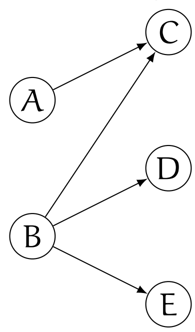
(iii) Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called Bow-Tie diagram of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.

(b) [**10** marks]

(i) Describe the Hubs and Authorities procedure for the ranking of web pages. What is its input, what is its output, and what sequence of steps is used to compute the output from the input?

(ii) Why does the Hubs and Authorities ranking procedure converge?

(iii) Show the values obtained by applying two rounds of the Hubs and Authorities ranking procedure to the following network of web pages.



Summer Examinations 2017/18

Exam Code(s)	4BS2, 4FM2, 4BCT1, 4BMS2, 4BME1, 1BMG1
Exam(s)	4th Science
Module(s)	Networks
Module Code(s)	CS4423
External Examiner(s)	Prof. T. Brady
Internal Examiner(s)	Prof. G. Ellis
	*Prof. G. Pfeiffer
Instructions	Answer *all* questions.
Duration	2 hours
No. of Pages	4 pages (including this cover page)
Discipline	Mathematics

Requirements:

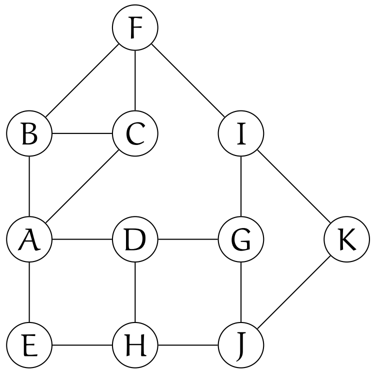
Release to Library	Yes ☑	No ☐
Release in Exam Venue	Yes ☑	No ☐
MCQ	Yes ☐	No ☑
Statistical/Log Tables	Yes ☐	No ☑

1.

(a) [**10** marks]

(i) Describe Breadth First Search as an algorithm for undirected graphs. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?

(ii) In the network on the side, apply Breadth First Search to determine a spanning tree with root D and the shortest distances from node D to each of the other nodes in the graph.



(b) [**10** marks] A graph, whose edges are all labelled either '+' or '−', satisfies the *Structural Balance Property*, if any triangle in the graph (3 nodes joined by 3 edges) has an odd number of edges (i.e., either 1 or all 3 edges) labelled '+' .

(i) What is a *complete* graph?

(ii) What does *Harary's Balance Theorem* say about a complete graph whose edges are all labelled either '+' or '−'?

(iii) Sketch a proof of Harary's Balance Theorem.

2.

(a) [**10** marks]

(i) Provide a formal definition of a finite n-player game.

(ii) What is a *best response* and what is a *Nash equilibrium* in a game?

(iii) Consider the following 2-player game, where the rows correspond to player A's strategies, and the columns to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

	L	R
U	3, 3	1, 2
D	2, 1	3, 0

Find all (pure) Nash equilibria of this game. Then change one but not both of the payoffs in the (U, L) entry in the above game to some nonnegative integer value, so that the resulting game has no pure Nash equilibrium. Compute a mixed-strategy equilibrium for the resulting game.

(b) [**10** marks] Two cities, A and B, are joined by two routes through distinct cities C and D, and 80 cars need to travel from A to B. The first route consists of a highway from A to a city C and a local road from C into B. The second route consists of a local road from A to a city D, and a highway from D to B. The travel time along a highway is one hour, regardless of the number of cars using it. The travel time along a local road is 10 plus the number of cars using it. Drivers simultaneously choose which route to use.

(i) Draw the transport network described above and label each edge with the time needed to travel along that edge. Let x be the number of cars travelling along the first route. Drivers simultaneously choose which route to use. Find the Nash equilibrium value for x.

(ii) The government builds a new two-way road connecting cities C and D. Describe the new routes from A to B which are added by this connection.

(iii) The new road is very short and takes no travel time at all. Find the new Nash equilibrium. What happens to the total travel time as a result of the availability of the new road?

3.

(a) [**10** marks]

(i) Define the term *bipartite graph*. How can a bipartite graph be characterized in terms of its cycles?

(ii) What does the Matching Theorem say about certain bipartite graphs?

(iii) Sketch a proof of the Matching Theorem.

(b) [**10** marks]

(i) Describe the concept of market clearing prices between a set of sellers and a set of buyers.

(ii) Suppose there are three sellers, A, B and C, and three buyers, X, Y and Z. Each seller is offering a house for sale, and the valuations of the buyers for the houses are as listed in the following matrix.

	A	B	C
X	3	6	4
Y	2	8	1
Z	1	2	3

Describe briefly the bipartite graph auction procedure. Apply this procedure to the above valuations in order to obtain a set of market clearing prices.

4.

(a) [**10** marks] Suppose that $G = (X, E)$ is a directed network with node set X and edges $E \subseteq X^2$.

(i) Define what it means for G to be *weakly connected*, and what it means to be *strongly connected*.

(ii) In general, a directed network is partitioned into strongly connected components. Describe the equivalence relation that yields strongly connected components as its equivalence classes in terms of E, regarded as a relation on X.

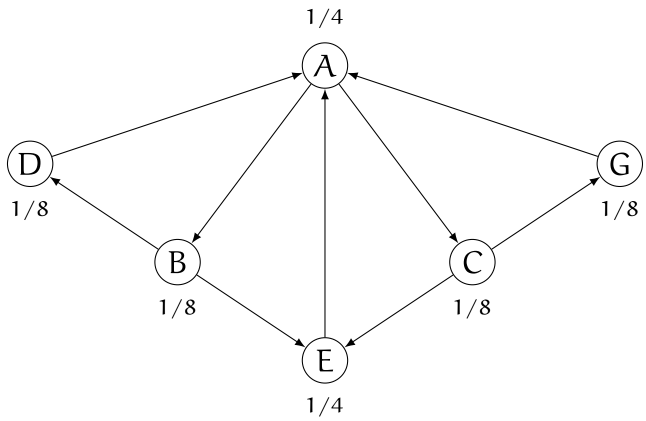
(iii) Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called Bow-Tie diagram of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.

(b) [**10** marks]

(i) Describe the PageRank algorithm. What is its input, what is its output, and what sequence of steps is used to compute the output from the input?

(ii) In this context, what is an equilibrium for PageRank?

(iii) In the network below, does the assignment of numbers to the nodes form an equilibrium? Justify your answer.



Summer Examinations 2018/19

Exam Code(s)	4BS2, 4BMS2, 4FM2, 4BCT1, 4BME1, 1SPA1, 4SPE1
Exam(s)	4th Science
Module(s)	Networks
Module Code(s)	CS4423
External Examiner(s)	Prof. T. Brady
Internal Examiner(s)	Prof. G. Ellis
	*Prof. G. Pfeiffer
Instructions	Answer *all* questions.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline	Mathematics

Requirements:

Release to Library	Yes	☑	No	☐
Release in Exam Venue	Yes	☑	No	☐
MCQ	Yes	☐	No	☑
Statistical/Log Tables	Yes	☐	No	☑

1.

(a) [**5** marks] What is a *graph*? What is a *directed graph*? What is a *tree*?

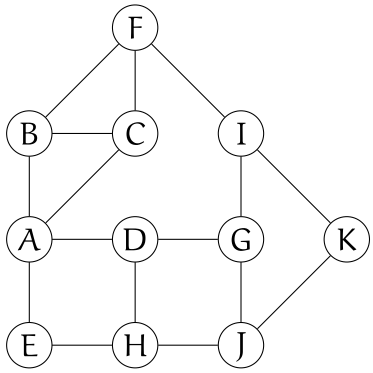
(b) [**5** marks] At a party with $n = 5$ people, some people know each other already while others don't. Each of the 5 guests is asked how many friends they have at this party. Two report that they have one friend each. Two other guests have two friends each, and the fifth guest has three friends at the party. Understanding friendship as a symmetric relation, is this network possible? Why, or why not?

(c) [**10** marks]

(i) Describe *Breadth First Search* as an algorithm for undirected graphs. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?

(ii) In the network on the side, apply Breadth First Search to determine a *spanning tree* with root G and the *shortest distances* from node G to each of the other nodes in the graph.

(iii) Use the Breadth First Search from part (ii) to also determine, for each node x, its possible *predecessors* on a shortest path from G to x. Use this information to list *all shortest paths* from G to B.



2.

(a) [**5** marks] Define the concepts of *degree centrality* and of *normalized degree centrality* for a graph G.

Let G be the graph on the vertex set $\{1, 2, 3, 4, 5, 6, 7\}$ with edges $1-2$, $1-3$, $2-4$, $2-5$, $3-4$, $4-6$, $5-6$ and $6-7$. For each node in this graph G, determine its normalized degree centrality. (A sketch of G might be useful.)

(b) [**5** marks] The fact that

$$\begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

makes $\lambda = 4$ an *eigenvalue* with *eigenvector* $v = \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right)$ of the matrix $A = \left(\begin{smallmatrix} 2 & 2 \\ 3 & 1 \end{smallmatrix}\right)$. The other eigenvalue of A is -1. With this example in mind, formulate the Perron–Frobenius Theorem on square, irreducible, nonnegative matrices. Define the concept of *eigenvector centrality* for a connected graph G.

(c) [**5** marks] Define the concepts of *closeness centrality* and *normalized closeness centrality* for a connected graph G. Give an example of a connected graph on 5 vertices with a node of normalized closeness centrality 1.

(d) [**5** marks] Define the concepts of *betweenness centrality* and *normalized betweenness centrality* for a connected graph G.

3.

(a) [**10** marks]

(i) Define the two Erdős–Rényi models, $\mathsf{G}(n, m)$ and $\mathsf{G}(n, p)$ of random graphs.

(ii) For a fixed value of n, describe each of the two models as a probability distribution on the set G_n of *all* graphs on the n nodes $X = \{1, 2, \ldots, n\}$.

(iii) For $n = 100$ and $p = \frac{1}{99}$, what is the probability that a graph G sampled from the $\mathsf{G}(n, p)$ model has exactly 50 edges?

(b) [**10** marks]

(i) Define the degree distribution of a graph G.

(ii) What is the degree distribution in a random graph G in the $\mathsf{G}(n, p)$ model? What is the expected average degree in such a graph G?

(iii) Construct a graph G on 100 nodes by throwing a (fair, 6-sided) dice once for each possible edge, adding the edge only if the dice shows a 6. Then pick a node at random in this graph. What is the probability that this node has degree 10?

4.

(a) [**10** marks]

(i) What is the *node clustering coefficient* of a vertex x in a graph G? What is the *graph clustering coefficient* C of G?

(ii) Determine the graph clustering coefficient C of a random graph in the $\mathsf{G}(n, p)$ model. How does C behave in the limit $n \to \infty$, when the average node degree is kept constant? What practical consequence does this observation have?

(iii) Describe the *Watts–Strogatz small-world model* (WS model). What properties does a random graph sampled from the WS model have, that one wouldn't find in a random graph sampled from the $\mathsf{G}(n, p)$ model, or in an (n, d)-circle graph?

(b) [**10** marks] Suppose that $G = (X, E)$ is a *directed* network with node set X and edge set E.

(i) Define what it means for G to be *weakly connected*, and what it means to be *strongly connected*.

(ii) In general, a directed network is partitioned into strongly connected components. Describe the equivalence relation that yields strongly connected components as its equivalence classes in terms of the edge set E, regarded as a relation on X.

(iii) Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called Bow-Tie diagram of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.

NUI Galway
OÉ Gaillimh

Summer Examinations 2021-2022

Exam Codes	1BMG1, 1GDS1, 3BA1, 4BCT1, 4BDS1, 4BME1, 4BMS2, 4BS2, 4FM2
Exam	4th Science
Module	Networks
Module Code	CS4423
External Examiner(s)	Prof Colva Roney–Dougal
Internal Examiner(s)	Dr N. Madden Dr A. Carnevale*
Instructions	Answer all **four** questions.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline	Mathematics

Requirements:

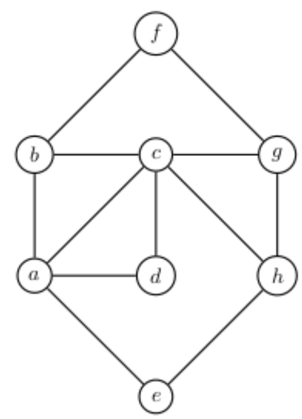
Release to Library	Yes
Release in Exam Venue	Yes
Statistical Tables/ Log Tables	Yes

Q1. [15 Marks]

(a) [3 Marks] What is a *graph*? What is a *tree*? Give an example of a graph of *order* 5 and *size* 6. Does a tree of *order* 5 and *size* 6 exist? Justify your answer.

(b) [5 Marks] What is the *Prüfer code* of a labelled tree and how can it be obtained? How can the degree sequence of a labelled tree be obtained from its Prüfer code? Which labelled tree on vertices $\{0, 1, \dots, 6\}$ has Prüfer code $[1, 1, 1, 1, 1]$? Justify your answer.

(c) [3 Marks] Describe *Breadth First Search* as an algorithm for computing distances between vertices in a graph. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?



(d) [4 Marks] In the network on the right, apply Breadth First Search to determine

(i) a spanning tree with root a, and

(ii) the shortest distances from vertex a to each of the other vertices in the graph.

Q2. [15 Marks]

(a) [5 Marks] Define the concepts of *degree centrality* and of *normalised degree centrality* for a graph G. Let G be the graph on the vertex set $\{0, 1, 2, 3, 4, 5, 6\}$ with edges $0-3$, $1-2$, $1-3$, $2-3$, $3-4$, $3-5$, $3-6$ and $4-5$. Draw the graph G and determine the normalised degree centrality of all its vertices.

(b) [5 Marks] Define the concepts of *closeness centrality* and *normalised closeness centrality* for a connected graph G. Give an example of a connected graph on 7 vertices with a vertex of normalised closeness centrality 1.

(c) [5 Marks] Define the concepts of *betweenness centrality* and *normalised betweenness centrality* for a connected graph G. Give an example of a connected graph on 7 vertices with a vertex of normalised betweenness centrality 0.

Q3. [15 Marks]

(a) [3 Marks] Describe how to generate a graph from the Erdős–Rényi model $G(n, m)$.

(b) [4 Marks] Describe how to generate a graph from the Erdős–Rényi model $G(n, p)$. Following this model, what is the probability of a randomly chosen graph G to have *exactly* m edges? Justify your answer.

(c) [4 Marks] Construct a graph G on 80 vertices by tossing a fair coin once for each possible edge, adding the edge only if the coin shows heads. Then pick a vertex at random in this graph. What is the probability that this vertex has degree 40?

(d) [4 Marks] What is a *giant component* in a graph G? State the *Erdős–Rényi Theorem* on the appearance of a giant component in a graph.

Q4. [15 Marks]

(a) [3 Marks] Sketch an example of an (n, d)-circle graph for $n = 8$ and $d = 2$. How many edges does this graph have?

(b) [5 Marks] Describe how to generate an (n, d, p)-WS graph in the *Watts–Strogatz small-world model*. What properties does a random graph sampled from the WS model have, that one would not find in a random graph sampled from the $G(n, p)$ model, or in an (n, d)-circle graph?

(c) [4 Marks] For a directed graph G on a vertex set X, define two equivalence relations on X: one that has the *strongly connected components* of G as its equivalence classes, and one that has the *weakly connected components* of G as its equivalence classes.

(d) [3 Marks] Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called *Bow-Tie diagram* of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.

Semester 2 Examinations 2023

Exam Codes	3BME1, 4BCT1, 4BDS1, 4BME1, 4BMS2, 4BS2, 4FM2
Exam	4th Science
Module	Networks
Module Code	CS4423
External Examiner(s)	Prof. Colva Roney–Dougal
Internal Examiner(s)	Prof. G. Pfeiffer Dr A. Carnevale*
Instructions	Answer all **four** questions.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline	Mathematics

Requirements:

Release to Library	Yes
Release in Exam Venue	Yes
Statistical Tables/ Log Tables	Yes
Non-programmable calculators	Yes

Q1. [25 Marks]

(a) [10 Marks]

(i) What is a *graph*?

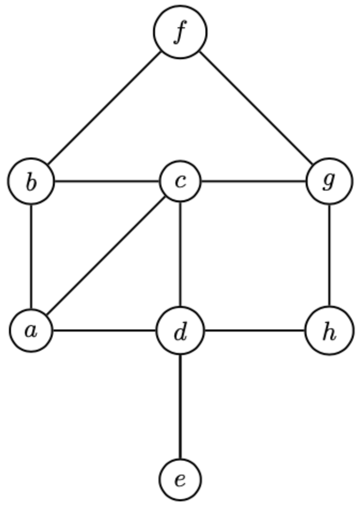
(ii) What are the *order* and the *size* of a graph?

(iii) What is the *adjacency matrix* of a graph?

(iv) How can one compute the degree of a vertex from the adjacency matrix?

(v) How can one compute the size of a graph from its adjacency matrix?

(b) [7 Marks] Describe *Breadth First Search* as an algorithm for computing distances between vertices in a graph. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?



(c) [4 Marks] Show how the Breadth First Search algorithm for distances proceeds when the inputs are the graph on the right and its node a.

(d) [4 Marks] In the graph on the right, use the Breadth First Search algorithm to also determine a *spanning tree* with root a.

Q2. [25 Marks]

(a) [5 Marks] Define the concept of *normalised degree centrality* for a graph G. Let G be the graph on the vertex set $\{1, 2, 3, 4, 5\}$ with edges $1-2$, $1-3$, $2-4$, $3-4$, $3-5$, and $4-5$. Draw the graph G and determine the normalised degree centrality of all its vertices.

(b) [5 Marks] Define the concept of *normalised closeness centrality for a connected graph G*. Compute the normalised closeness centrality of nodes 1 and 2 of G in (a).

(c) [5 Marks] Define the concept *normalised betweenness centrality* for a connected graph G. Compute the normalised betweenness centrality of nodes 1 and 2 of G in (a).

(d) [10 Marks] Let H be a path graph on 3 vertices. The eigenvalues of the adjacency matrix of A are $-\sqrt{2}$, 0 and $\sqrt{2}$. Use this information to compute the normalised eigenvector centrality of each node in H.

Q3. [25 Marks]

(a) [8 Marks] Describe how to generate a graph from the Erdős–Rényi models $G(n, m)$ and $G(n, p)$. In each model, what is the probability of a randomly chosen graph G to have *exactly* m edges? Justify your answer.

(b) [5 Marks] Suppose one constructed a graph G on 100 vertices by tossing a (fair, 6-sided) die once for each possible edge, adding the edge only if the die shows 6. Then pick a vertex at random in this graph. What is the probability that this vertex has degree 50? (You don't need to return a numerical value. It is enough to give an explicit formula in terms of the given data.)

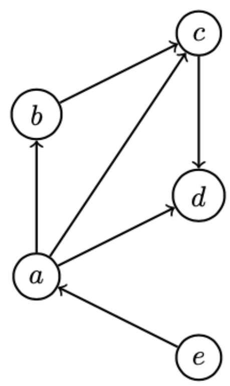
(c) [5 Marks] What is a *giant component* in a graph G? State the *Erdős–Rényi Theorem* on the appearance of a giant component in a graph.

(d) [7 Marks] Describe how to generate an (n, d, p)-WS graph in the *Watts–Strogatz small-world model.* What properties does a random graph sampled from the WS model have, that one would not find in a random graph sampled from the $G(n, p)$ model, or in an (n, d)-circle graph?

Q4. [25 Marks]

(a) [8 Marks] What is the *Prüfer code* of a labelled tree T on n vertices, and how can it be computed from T? How can the degree sequence of a labelled tree be determined from its Prüfer code? Compute the Prüfer code of the tree on the vertex set $X = \{1, 2, 3, 4, 5, 6\}$ with edges 1–2, 2–3, 3–4, 4–5, 4–6.

(b) [5 Marks] What is a *directed graph*? What is the in-degree and what is the out-degree of vertex c in the directed graph on the right?



(c) [7 Marks] For a directed graph G on a vertex set X, define two equivalence relations on X: one that has the *strongly connected components* of G as its equivalence classes, and one that has the *weakly connected components* of G as its equivalence classes.

(d) [5 Marks] Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called *Bow-Tie diagram* of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.

OLLSCOIL NA GAILLIMHE
UNIVERSITY OF GALWAY

Semester 2 Examinations 2024

Exam Codes	3BA1, 4BDS1, 3BME1, 4BME1, 4BCT1, 1BMG1, 4BMS2, 4BS2, 4FM2, 1GDS1
Exam	4th Science
Module	Networks
Module Code	CS4423
External Examiner(s)	Prof. Colva Roney–Dougal
Internal Examiner(s)	Dr N. Madden Dr A. Carnevale*
Instructions	Answer all **four** questions.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline	Mathematics

Requirements:

Release to Library	Yes
Release in Exam Venue	Yes
Statistical Tables/ Log Tables	Yes
Non-programmable calculators	Yes

Q1. [25 Marks]

(a) [10 Marks]

(i) What is a *graph*?

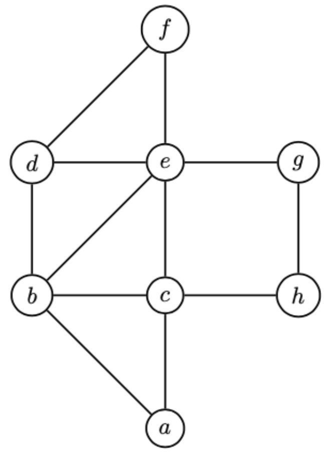
(ii) What are the *order* and the *size* of a graph?

(iii) What is the *adjacency matrix* of a graph?

(iv) Let A be the adjacency matrix of a graph G. Explain how one can compute the size of G as a function of the entries of A.

(v) Let A be the adjacency matrix of a graph G. Explain how one can compute the degree of the nodes of G as a function of the entries of A^2.

(b) [4 Marks] In the graph on the right, use the Breadth First Search algorithm to determine all the shortest distances between a and all the other nodes.



(c) [7 Marks] In the graph on the right, use the Breadth First Search algorithm for predecessors to determine all the predecessors in a shortest path from a to all the other nodes. Use this information to construct all the shortest paths between nodes a and f.

(d) [4 Marks] Determine a spanning tree of the graph on the right by using the Breadth First Search algorithm for spanning trees with inputs the graph itself and its node a.

Q2. [25 Marks]

(a) [5 Marks] Let G be the graph on the set of nodes $\{1, 2, 3, 4, 5, 6\}$ with edges $1-2$, $1-3$, $2-4$, $3-4$, $3-6$, $4-5$, $4-6$. Draw the graph G and determine the normalised degree centrality of all its nodes.

(b) [5 Marks] Define the concept of *normalised closeness centrality for a node in a connected graph* G. Compute the normalised closeness centrality of node 1 of G in (a).

(c) [5 Marks] Compute the normalised betweenness centrality of node 5 of G in (a). Justify your answer.

(d) [3 Marks] Is the graph G in (a) bipartite? Justify your answer.

(e) [7 Marks] Let H be a complete graph on 3 nodes. The eigenvalues of the adjacency matrix of A are 2 (with multiplicity 1), and -1 (with multiplicity 2). Use this information to compute the normalised eigenvector centrality of each node in H.

Q3. [25 Marks]

(a) [5 Marks] Describe how to generate a graph from the Erdős–Rényi models $G(n, m)$ and $G(n, p)$. In each model, what is the probability that a randomly chosen graph G has *exactly* m edges? Justify your answer.

(b) [5 Marks] For which value of p does sampling graphs from the $G(n, p)$ model yield an expected size m?

(c) [5 Marks] Suppose one constructed a graph G on 120 nodes by tossing a (fair, 6-sided) die once for each possible edge, adding the edge only if the die shows 3 or 6. Then pick a node at random in this graph. What is the probability that this node has degree 50? (You do not need to return a numerical value. It is enough to give an explicit formula in terms of the given data.)

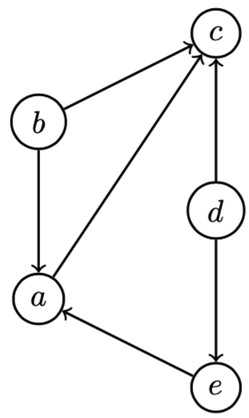
(d) [5 Marks] Describe how to generate an (n, d, p)-WS graph in the *Watts–Strogatz small-world model* from an (n, d)-circle graph.

(e) [5 Marks] How do the clustering and characteristic path length of a random graph sampled from the WS model relate to those in a random graph sampled from the $G(n, p)$ model, or in an (n, d)-circle graph?

Q4. [25 Marks]

(a) [8 Marks] Compute the Prüfer code of the tree on the set of nodes $X = \{1, 2, 3, 4, 5, 6\}$ with edges 1–2, 1–3, 1–4, 4–5, 5–6. Explain how the degree sequence of this tree can be determined from its Prüfer code.

(b) [5 Marks] What is a *directed graph*? What is the in-degree and what is the out-degree of node a in the directed graph on the right?



(c) [7 Marks] For a directed graph G on a set of nodes X, define an equivalence relation on X that has the *strongly connected components* of G as its equivalence classes. Explain why the graph on the right has 5 strongly connected components, each consisting of a single node.

(d) [5 Marks] Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called *Bow-Tie diagram* of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.