Game theory

Reasoning about Interactions:

Assume we have just two agents, *i* and *j*.

Assume agents are self-interested.

Let there be a set of “outcomes‟:

$$\Omega = \{\Omega_1, \Omega_2, \ldots \Omega_n\}$$

over which the the agents have preferences.

Preferences are expressed by utility functions:

$$u_i: \Omega \rightarrow \mathbf{R}$$

$$u_j: \Omega \rightarrow \mathbf{R}$$

These functions lead naturally to preference orderings over outcomes:

$$\Omega >=_i \Omega' \quad \rightarrow \quad u_i(\Omega) >= u_i(\Omega')$$

We need to model of the environment in which agents can act.

Let us assume agents act simultaneously to choose an action to perform, and as a result of the actions an outcome will result. The actual outcome depends on the combination of actions.

Can be represented as a state transformation function:

$$\tau: \text{Action}_i \times \text{Action}_j \rightarrow \Omega$$

(E.g. Consider simultaneous move games)

We will (for the time being) make the simplifying assumption that an agent can make one of two actions (to cooperate(C) or to defect(D)).

We say a certain move is *rational* if the outcomes that arise through that action are better than all outcomes that arise from the alternative action.

		Player j	
		C	**D**
Player i:	**C**	(1, 1)	(1, 4)
	D	(4, 1)	(4, 4)

For player j, **D** is the rational choice.

Dominant strategy

Given a particular strategy *s*, for agent *i*, there will be a number of possible outcomes.

We say *s1* dominates *s2*, if every outcome possible by agent *i* playing *s1* is preferred over every possible outcome by agent *i* playing s2.

A rational agent will never play a dominated strategy.

However, there is not usually a unique undominated strategy.

Nash Equilibrium

Two strategies *s1* and *s2* are in Nash equilibrium if:

assuming agent i plays *s1*, agent *j* can not do better than play *s2*;

and

assuming agent *j* plays *s2*, agent *i* can not do better than play *s1*.

In Nash equilibrium, neither agent has any incentive to deviate from their strategy.

Not all possible interactions have a Nash equilibrium.

Some interactions can have several Nash equilibria.

Prisoner’s Dilemma: Usually expressed in terms of pay-offs or rewards) for cooperating or defecting:

		Player j	
		C	**D**
Player i:	**C**	(3, 3)	(0, 5)
	D	(5, 0)	(1, 1)

If both cooperate; get a reward of 3. If both defect; get a reward of 1.

If one cooperates and the other defects, the cooperator gets 0 (the sucker's payoff) and the other gets 5.

The individually rational action is to defect. Guarantees a payoff of no worse than 1, whereas cooperating guarantees a payoff of no worse than 0.

So defection, is the best response to all strategies.

But, common sense indicates that it is *not* the best response.

This *prisoner's dilemma* occurs in many domains and is suitable for modelling large classes of multi-agent interactions.

There have been many real-world scenarios that are implicitly prisoner’s dilemmas (or variations):

- arms race,
- environmental issues,
- free-rider systems,
- warfare,
- behaviour in many biological systems - bats, guppie fish,
- competition between nodes in a distributed computer system,
- modelling competition and collaboration between information providers
- sports

Variations:

N-player dilemma: e.g. Voter's paradox, where it is true that a particular endeavour would return a benefit to all members where each individual would receive rewards, it is also true that any member would receive an even greater reward by contributing nothing". Elections, environmental actions and the tragedy of the commons are all examples of this phenomenon.

Spatial organisations: where agents are placed in some 2 dimensional space and can only interact with neighbours. (e.g Epstein, Lindberg, Haurt)

(evoplex.org)

Partial Cooperation: acts are no longer cooperative of non-cooperative, but can be in some range. We consider extending the classical IPD to this domain, we can define landscapes using pay-off equations:

Noise: Problems arise if we introduce any degree of noise, which will lead cooperations to be interpreted as defections etc. Consider two tfts playing with a degree of noise.

Summary so far

Need means to organise and coordinate agents. Underlying problems with respect to cooperation.

Game theory and extensions provides a tool to reason about and to develop multi-agent systems.

We assume agents have a rational ordering of possible outcomes and a set of actions they may choose to bring about those outcomes.

Have limited the types of interactions to very simple cases

Extensions - Ultimatum game:

No longer just discussing outcome for simple choices

Two players i and j.

Goal is to distribute some resource. Eg. 100 euro.

Player 1 picks a number, x, in a range (0-100).

Player 2 must accept or reject offer.

If Player 2 rejects; both get 0

If Player 2 accepts, Player 1 gets x, Player 2 gets 100-x

Allows us to reason about more complex scenarios.

Many extensions available/researched

If we wish to reason about two or more agents/systems agreeing on a value for some exchange (information, service), can look to auction theory.

To reason about more complex scenarios – negotiation and argumentation theory have been adopted

Auctions - introduction

Can be used as a method to allow agents arrive at an agreement regarding events and actions when agents are self-interested.

In some cases no agreement is possible at all.

However, in most scenarios, there is the potential to arrive at a mutually beneficial agreement.

There are several approaches that been adopted. All can be seen as a form of negotiation or argumentation by the agents.

Negotiation or argumentation are governed by some protocol (mechanism).

This protocol defines how the agents are to interact, i.e., the actual rules of encounter.

Questions that arise:

how to design a protocol such that certain properties exist?

how to design strategies for agents to use given a set of protocols?

Protocols

guaranteed success

simplicity

maximising social utility

pareto-efficiency

individual rationality

Auctions

Auctions represent a class of useful protocols. Used in many domains.

An auction takes place between an agent (auctioneer) and a set of other agents (bidders).

The goal is to allocate the *goods* to one of the bidders.

Usually auctioneer attempts to maximise the price; the bidders desire to minimise the price.

Can categorise auctions according to a range of features:

Bids may be:

  - open-cry
  - sealed bid

Bidding may be:

  - one shot
  - ascending
  - descending

Selling goods by auction is more flexible than setting a fixed price and less time-consuming that explicit negotiation (haggling).

In many domains, the value of a item may vary enough to preclude direct and absolute pricing.

It is a pure form of market. It is efficient in that auctions usually ensure goods are allocated to those who value them most.

The price is set, not by the sellers, but by the buyers.

No one auction protocol is best.

Some auctions are preferred by sellers, others by buyers.

Some auctions attempt to prevent cheating, or at least decrease the incentive to cheat; others provide several means to cheat.

People tend to bid in auctions for two reasons:

they wish to acquire the goods (bases bid on private evaluation)

they wish to acquire the goods to resell (bases bid on private evaluation and estimates on future valuations)

English Auction

The auctioneer begins with the lowest acceptable price (reserve), and proceeds to obtain successively higher bids from bidders until no-one will increase the bid.

It is effectively first-price, open-cry and ascending.

The dominant strategy is to successively bid a small amount more than the current highest bid until it reaches their valuation; then withdraw.

Potential problems:

rings

shills in the bidders

winner's curse

In some English auctions, the reserve price is kept secret to attempt to prevent rings from forming.

Dutch auction

Bidding starts at an artificially high price.

Lower prices are offered, in descending order, until a bidder equals to the current price.

Goods sold to that bidder for that price.

A descending, open-cry auction.

From a seller's perspective, the key to a successful auction is the effect of competition on the bidders. In an English auction, a winner may pay well under his/her valuation and thus the seller loses out.

This is not the case in a Dutch auction.

First-Price, Sealed Bid

Usually one-shot auctions. Each bidders submits a sealed bid. The goods are sold to the highest bidders.

Best strategy is to bid to true valuation.

Interesting variations exist if there are a number of goods to be sold and a number of rounds.

Vickrey Auction

This is a sealed-bid, second-price auction. The price paid by the winner is that price offered by the second placed bidder.

In this type of auction, contrary to initial intuition, seller's make as much if not more than first-price auctions.

In reality, bidders are not afraid to bid high, knowing they will have to pay the second price; bidders tend to be more competitive.

Other auction types exist also: reverse auctions, double auctions, haphazard (whisper auction, handshake auction)…

We can use auctions are a means to allow agents agree on a price for buying goods or services. Depending on the type of auction chosen, we will favour buyers or sellers.

We still have some problems though:

Are auctions the best way?

What happens following an auction, if upon receiving goods one doesn't pay.

What happens following an auction, if upon paying, one realises the goods are not as expected.

Is it possible to prevent shills, rings and other forms of manipulation?

In auctions, agents agree on a price; can we deal with more dimensions of negotiation?