CS4423: Problem Set 2

These exercises should help you prepare for the class test, which will be somewhat similar in structure:

- Q1 will have 10 "true/false" based on material covered up to, and including Week 7.
- Three other questions, again on any material up to and including Week 7.

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Q1. For each of the following, state whether it is **true** or **false**. Explanations are not required. In all cases G represents a graph: $G = (X, E)$ with node set X, and edge set E.

(i) The **order** of G is $|E|$.

(ii) The **degree** of a node is the number of times it occurs in X

(iii) A bipartite graph is two-colourable.

(iv) The path graph on n nodes, P_n, is a tree.

(v) Let G_1 be the graph on the set of nodes $\{0, 1, 2, 3, 4\}$ with edges $0-1$, $0-2$, $0-3$, $1-4$, $2-3$. G_1 is isomorphic to its complement.

(vi) G_1, the graph in the previous question, has the same order as its line graph.

(vii) The adjacency matrix of a digraph cannot be symmetric.

(viii) There exists a 5×5 adjacency matrix with Perron Root $\lambda = 2$, and corresponding eigenvalue $v = (1, -1, 1, -1, 1)$.

(ix) $a = (4, 3, 2, 1, 4)$ is a valid Prüfer code for a tree with nodes $\{0, 1, 2, 3, 4, 5, 6\}$.

(x) The cycle graph on n nodes, C_n, has diameter $\lceil n/2 \rceil$, where $\lceil \cdot \rceil$ is the *ceiling* function.

Q2. Consider the following matrix:

$$A_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{1}$$

(a) Give a sketch of the graph, G_2, on the nodes $X = \{0, 1, 2, 3, 4\}$ with the that has A_2 as its adjacency matrix.

(b) Is this graph bipartite? If so, indicate a two-colouring in your sketch.

(c) Give the *relative degree centrality* of the nodes in G_2.

(d) A_2 has as an eigenvector $v = (2, 1, a, b, c)$. Compute a, b and c, as well as the eigenvalue that corresponds to this eigenvector.

(e) Compute A_2^2 (Note: this can be done either by matrix multiplication, or just looking at the graph. Either approach is fine). Verify that $A_2 + A_2^2 > 0$. What is the implication of that for the diameter of G_2?

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Q3. (a) Sketch the tree, G_3, on the nodes $\{0, 1, 2, 3, 4, 5, 6\}$ with edges $0-1$, $1-2$, $1-3$, $1-4$, $3-5$, $3-6$.

(b) Compute the Pruefer code for G_3.

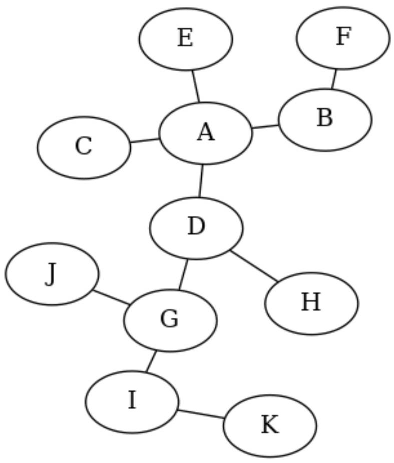
(c) Determine the tree on the nodes $\{0, 1, 2, 3, 4, 5\}$ which has Pruefer code $(1, 2, 1, 3)$.

Q4. Consider the graph T_4 and G_4 shown in Figure 1a.

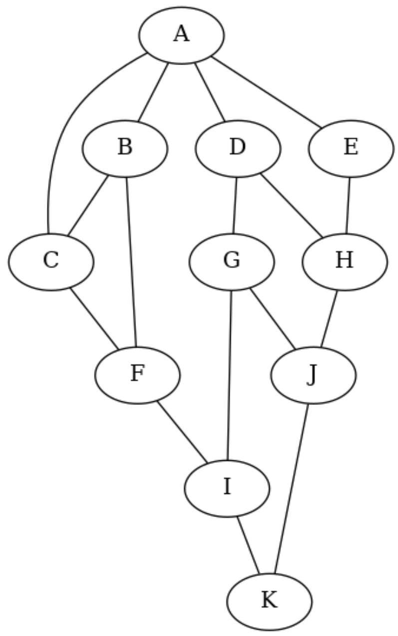
(a) List the nodes of T_4 in the order they would be traversed by the **depth-first search** (DFS) algorithm, starting at node A.

(b) List the nodes of T_4 in the order they would be traversed by the **breadth-first search** (BFS) algorithm, starting at node A.

(c) For the graph G_4, apply the BFS algorithm to determine the distances from node A to all other nodes in the graph.



(a) T_4: for Q4(a) and (b)



(b) G_4: for Q4(c)

Figure 1: Graphs for Q4