MA284 : Discrete Mathematics

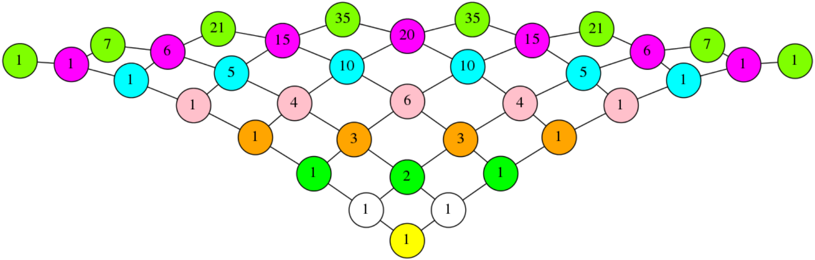
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

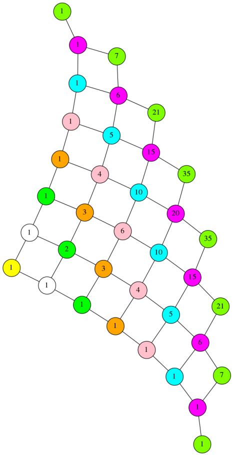
delivered by Kevin Jennings (Kevin.Jennings@UniversityOfGalway.ie)

with thanks to Dr Niall Madden who prepared the material and notes

Any mistakes or typos are Kevin's ... any bad jokes have multiple parents ...

7 & 9 September, 2022

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MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 1: All about MA284

Lecturer: Dr Kevin Jennings (he/him),
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Email: Kevin.Jennings@UniversityOfGalway.ie,

The best way to contact me is by email (or possibly using the chat feature on Blackboard).

https://commons.wikimedia.org/wiki/File:%C3%81ras_de_Br%C3%BAn.jpg

This module is taken by about 224 (and counting) students in

- 2nd Science: Mathematics, Mathematical Science, Physics, E&O, Computer Science, Financial Mathematics and Economics, ...;
- Arts: 2nd Mathematics, 2nd Music(!), 3rd Mathematics/Computer Science & Education, Data Science...;
- 2nd Computer Science & IT (2BCT1);
- Visiting student(s).

Given your *very* varied backgrounds, you will need to stay focused, and become practised at communicating your own insights and challenges...

This is *Discrete Mathematics*: a mathematics module introduces the concepts of

- *enumerative combinatorics*: how to count,
- *graph theory* (i.e., the theory of graphs).

Don't worry: most of the rest of the definitions in this module will be more helpful than that!

Lectures: Wednesdays, **13.00–13.50** Anderson Theatre
Fridays, **11.00–11.50** Fottrell Lecture Theatre.

Tutorials: They will start in Week 3. More details in a moment.

Blackboard: You will find lots of resources on Blackboard

- Announcements;
- These slides;
- Grades;
- Link to the textbook,
- Access to assignments.

Work load: 5 ECTS (60 is the typical total for a full-time programme)
24 lectures, all in Semester 1
Roughly 120 hours of student effort time.

Lecture materials: Slides for the week's classes will be available for download in advance of the Wednesday lecture. Please let me know if you spot typos or if the slides are inaccessible in any way (eg to screen-readers).

The slides contain the main definitions, ideas, and examples. Examples that are worked out in class will be posted later in the week.

Each set of slides finishes with a list of exercises, which are of a similar style and standard as those on the final exam.

Images: Particularly in the second half of this module, there will be lots of pictures of graphs. These are mostly generated using **Graphviz** `http://www.graphviz.org/` and/or NetworkX `https://networkx.github.io/`

I'll make Dr Madden's source code available. But if I forget, please ask!

SUMS: The School of Maths provides a free drop-in centre called

SUMS: Support for Undergraduate Maths Students.

SUMS opens from **2pm to 5pm, Monday to Friday**, from Monday of Week 3. For more information, see
http://www.maths.nuigalway.ie/sums/

Devices: The use of portable electronic devices during class is *encouraged*. For example, you might want to use it to check Wikipedia, or access the textbook.
Be aware that these can be distracting to other students. Please be considerate.

Other stuff: Last year this lecture fell on **Soc's Day**! Why not (re)join the Mathematics Society?
https://www.facebook.com/MathsSocNUIG
Also, consider joining our Student Chapter of SIAM:
http://www.maths.nuigalway.ie/SIAM-Galway/

MA284 will be assessed as follows.

Continuous assessment: There will be **five** online assignments, together worth 40% of the final grade.
Multiple attempts can be made, and scoring (right/wrong) is provided immediately. These will help you test yourself, and give you time to seek support at tutorials.

WeBWorK: The Online Assignments uses "**WeBWorK**", the same system as the interactive exercises in the text-book (more about that in a minute). You access the assignments through Blackboard.

Final assessment: There will be a 2 hour exam at the end of the semester, worth **60%**.

Tutorials will start in Week 3 (week beginning **19** September). You should attend *one tutorial per week*.

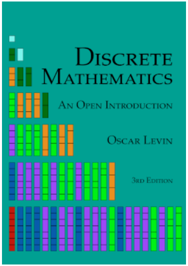
The tentative arrangements for this year below.

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11					
11 – 12			MRA 201		(Lecture)
12 – 1		CA117			
1 – 2			(Lecture)		
2 – 3			AC215		
3 – 4		AC213		ADB1020	
4 – 5				AMB-G008	

Please email Kevin now if *none* of these times work for you, with your course details.

The main recommended text is

Oscar Levin, Discrete Mathematics: an open introduction, 3rd Edition. This is a free, open source textbook, available from `http://discretetext.oscarlevin.com`, in both printable and tablet/ereader-friendly versions. It is published under Creative Commons (CC BY-SA 4.0)



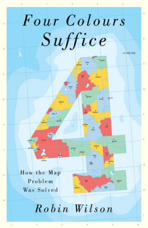
Other recommended texts include:

- Normal L Biggs, *Discrete Mathematics*, Oxford Science Publications. There are about 10 copies in the library at `510 BIG`.
- Kenneth Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill. Located at `511 ROS`.

Other books and resources will be mentioned through the semester.

Some related, fun, reading.

Really Big Numbers, by Richard Schwartz, published by the American Mathematical Society.
It is aimed a children, but is quite sophisticated. So you can learn some Discrete Mathematics while doing bed-time reading! It is in the library at 513 SCH
Watch at
https://www.youtube.com/watch?v=cEOY9UAsCFM



Four Colors Suffice: How the Map Problem Was Solved.
Robin Wilson.
In the library at 511.5 WIL
This is the story of the solution of one of most famous mathematical problems, that defied solution for nearly 150 years. It is also a treatise on what "proof" really means.

Do you have any other suggestions?

There are very few prerequisites for this module. I will expect that

- you can reason logically;
- understand the concept of a *proof*, and know several proof techniques, such as *induction*.
- know what a matrix is, and how to multiply a matrix by a vector, and a matrix by a matrix.
- you are comfortable with the concept of **sets**, and the notation used to describe and manipulate them.
- you are comfortable with the concept of **functions**, and the notation used to describe and manipulate them.

Exercise

Read Sections 0.3 (Sets) and 0.4 (Functions) in Chapter 0 of *Discrete Mathematics: an open introduction*

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 1

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 2: What is Discrete Mathematics

If calculus is "continuous mathematics", then "discrete mathematics" is everything else! However, it is usually taken to include the following

1. Logic
2. Sets and set-theory;
3. Mathematics of Algorithms;
4. Recursion and induction;
5. Counting;
6. Discrete probability;
7. Graphs, trees and networks;
8. Boolean algebra;
9. Modelling computing (Turing machines and Finite State Machines).

But we will just focus on **counting (combinatorics)** and *graphs*.

1. Combinatorics.

How to count,

The additive and multiplicative principles.

The Binomial coefficients and some identities.

The Principle of Inclusion-Exclusion.

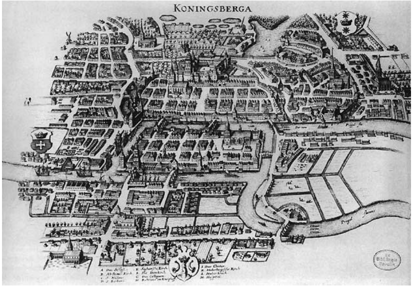
Permutations and Combinations.

Non-negative equations and inequalities.

Derangements and distributions.

2. Graph Theory.

Euler and the Koenigsberg Bridges Problem.
Eulerian and Hamiltonian graphs.
Tree graphs and bipartite graphs.
Planarity of Graphs.
Euler's formula for a connected planar graph.
Planarity and the Platonic solids.
Colouring of Graphs.



Combinatorics has an ancient history. The earliest known is in a 3,500 year old Egyptian manuscript. It posed a question like *"In 7 houses are 7 cats, each with 7 mice, who each have 7 heads of wheat, which each have 7 grains. How many houses, cats, mice, heads of wheat and grains are there?*

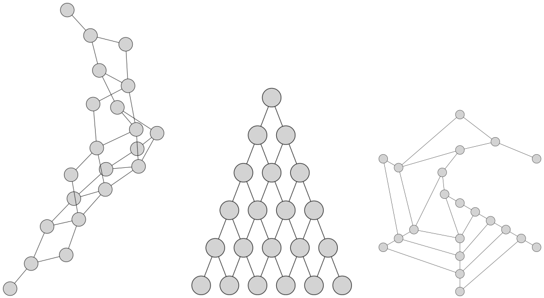
Description: The so-called "Rhind Mathematical Papyrus" : detail (British Museum, EA10057)

Source: `http://www.archaeowiki.org/Image:Rhind_Mathematical_Papyrus.jpg`

Slightly more recently, in the 6th century the Indian physician Sushruta determined that there are $2^6 - 1 = 63$ different combinations of the tastes *sweet*, *pungent*, *astringent*, *sour*, *salt*, and *bitter*.

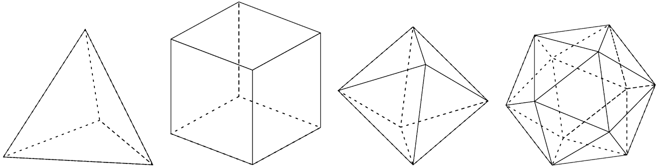
We'll solve problems like the two above, and also:

1. What are your chances of winning the Irish Lottery ("Lotto"). That is, what is the probability of correctly selecting **6** numbers from **47**?
2. If 500,000 people play the Lotto per week. What is the chance of a roll-over (i.e., nobody winning)?
3. For last night's men's European soccer match between Glasgow Celtic and Real Madrid, a **23**-man squad was named for Celtic.
   How many different ways were there of selecting the 11 starting players for the match?
   How many ways could one select (up to 5) of these players to be substituted during the game?
4. My password has 10 characters. Each character is an upper- or lower-case letter, or a digit. How long would it take you to crack my account?

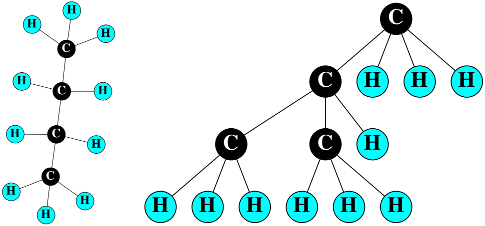


1. Which of these graphs are the **same** (and what does that mean)?
2. Is it possible to draw all the graph on the left so that none of its edges intersect?
3. What is the smallest number of colours needed to colour every vertices so that no two adjacent vertices have the same colour?
4. Is there a “route” through the graph that visits every vertex once and only once?

5. How may regular polyhedra (platonic solids) are there?



6. Are all the graphs of *saturated hydrocarbon isomers* trees?



The most important reason for taking this module is that **Discrete mathematics is one of the most appealing, elegant, and applicable areas of mathematics**.

Appealing: The problems that we will consider are, I believe, easily motivated, but not trivial.

Elegant: The solutions to these problems involve some clever reasoning, but never tedious calculations.

Applicable: In spite of its classical origins, graph theory is one of the hottest topics in both pure and applied mathematics, with applications to network science, computer science, linguistics, chemistry, physics, biology, social science, and music.

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 2

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 3: Counting

Combinatorics is the mathematics of *counting*. It is an ancient field of study, though its "modern" history began with the systematic study of gambling in the 17^{th} century.

The simplest method of counting is *simple enumeration* = *"Point and count"*.

1. How many students in this class have a last name that begins with `A`?

2. How many anagrams are there of the letters **`NUI`**?

   Usually we don't want to make a list of all possibilities:

3. How many car licence plates are there of the form `XXX-yyy`, where X is a letter and y is a digit?
   Answer: There are 17,576,000, but we don't want to list them all.

The first techniques that we will study for solving counting problems are called

The Additive and Multiplicative Principles

For more information see Chapter 1 (Counting) of Oscar Levin's *Discrete Mathematics: an open introduction*.

1. There are 5 starters and 6 main-courses on a restaurant's menu. How many choices do you have if
   (a) You would like one dish: a starter *or* a main-course?
   (b) You would like two dishes: a starter **and** a main-course?

2. A standard deck of cards has 26 red cards, and 12 face/court cards.
   (a) How many ways can you select a card that is red *and* face card?
   (b) How many ways can you select a card that is red *or* face card?

Think about these questions as we go through the next sections.

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 3

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 4: The Additive Principle

Example

The University of Galway Animal Shelter has 4 cats and 6 dogs in need of a home. You would like a new pet (but just one!). How many choices do you have?

The Additive Principle

If event A can occur m ways, and event B can occur n (disjoint) ways, then event "A **or** B" can occur in $m + n$ ways.

Example

1. Can we use the additive principle to determine how many two letter "words" **begin with** either A **or** B?

2. Can we use the additive principle to determine how many two letter "words" **contain** either A **or** B?

The Additive Principle

If event A can occur m ways, and event B can occur n (disjoint) ways, then event "A **or** B" can occur in $m + n$ ways.

Example

The University of Galway Animal Shelter has 4 cats, 6 dogs, and 7 donkeys in need of a home. How many choices do you have for a new pet?

The Additive Principle

If event A can occur m ways, and event B can occur n **disjoint** ways, then event "A **or** B" can occur in $m + n$ ways.

Example

A deck of cards has 26 red cards and 12 "face"-cards.

1. How many ways can you pick a red card?
2. How many ways can you pick a face-card?
3. How many ways can you pick a card that is red *or* is a face-card?

This last example is important because it emphasises the importance of the sets being **disjoint**.

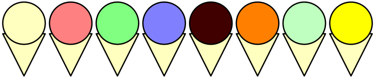
MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 4

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 5: The Multiplicative Principle



Example

Your favourite ice-cream shop has **8** flavours of ice-cream.
You can also choose between a cone, a waffle, and a cup.
How many choices to you have?

The Multiplicative Principle

If event A can occur m ways, and each possibility allows for B to occur in n (disjoint) ways, then event "A **and** B" can occur in $m \times n$ ways.

Example

The University of Galway Animal Shelter has 4 cats and 6 dogs in need of a home. How many choices do you have if you would like a cat and a dog as pets?

Example

The University of Galway Animal Shelter also has 7 donkeys. How many choices to you have if you want a cat, a dog and a donkey?

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 5

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of …

PART 6: Counting with sets

A set is a collection of things. The items in a set are called *elements*.
Examples:

- The set of natural numbers from 1 to 10 is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- The set of upper-case letters is $\{A, B, \ldots, Y, Z\}$
- The set of students registered for Discrete Mathematics has 224 elements.
- A set is *unordered*.

You should be familiar with the following basic elements of set notation:

$$\{\cdot\} \quad \in \quad \notin \quad \subseteq \quad \cup \quad \cap \quad \emptyset \quad |\cdot| \quad \backslash$$

Example

Let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$, and $C = \{2, 4\}$.

- $2 \in A$, $4 \notin A$
- $\{1, 3\} \subseteq A$
- $A \cup B = \{1, 2, 3, 5\}$
- $A \cap B = \{1, 3\}$, $B \cap C = \emptyset$,
- $|A| = 3$, $|B \cap C| = 0$,
- $A \backslash B = \{2\}$ $A \backslash C = \{1, 3\}$

Also, for any set X, $X \subseteq X$ $\emptyset \subseteq X$.

Let's return to the restaurant problem again, changed slightly...

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 6

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 7: Exercises

Here are a set of exercises to help you work through the material presented during this week's classes.

All but the last are taken either directly from the textbook, or with minor edits.

You do not have to submit your solutions to be graded.

0. Read Chapter 0 of Levin's *Discrete Mathematics: an open introduction* from `http://discretetext.oscarlevin.com`.
   Do Exercises 1–9 in Chapter 0 (these are interactive, with hints and solutions).
1. Your wardrobe consists of 5 shirts, 3 pairs of pants, 17 bow ties, and one fez (hat). How many different outfits can you make?
2. For your job interview at the University of Galway Animal Shelter, you must wear a tie. You own 3 regular (boring) ties and 5 (cool) bow ties. How many choices do you have for your neck-wear?

3 You realise that the interview is actually for ClownSoc, so you should probably wear both a regular tie and a bow tie. How many choices do you have now?

4 Your DVD collection consists of 9 comedies and 7 horror movies. Give an example of a question for which the answer is:
   (a) 16.
   (b) 63.

5 If $|A| = 10$ and $|B| = 15$, what is the largest possible value for $|A \cap B|$? What is the smallest? What are the possible values for $|A \cup B|$?

6 If $|A| = 8$ and $|B| = 5$, what is $|A \cup B| + |A \cap B|$?