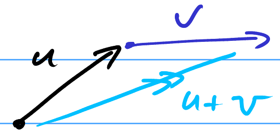
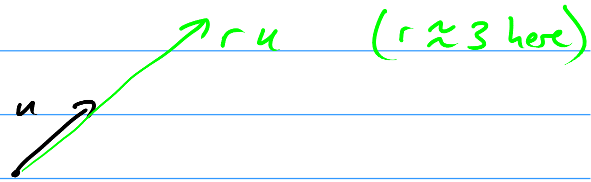
MA203 Lecture 2. Intro to Geometry/Vectors ctd.

Recap: Vectors in $\mathbb{R}^n$. Let $u, v \in \mathbb{R}^n$, $r \in \mathbb{R}$

- Can add $u+v$

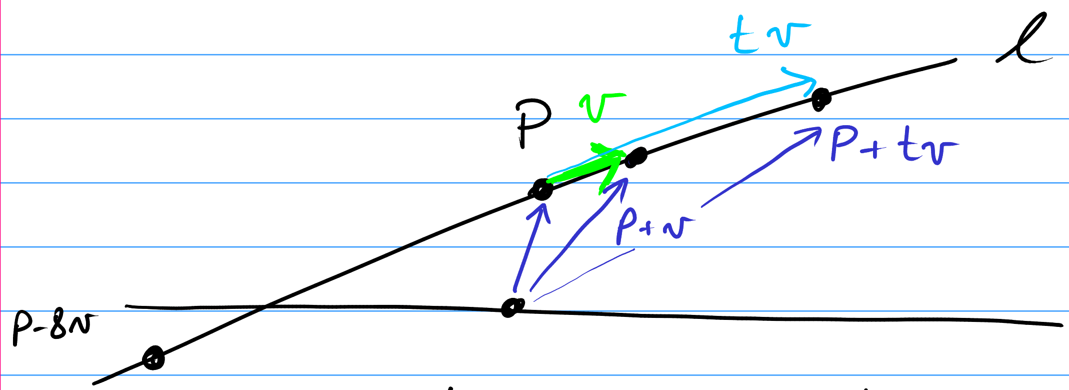


- Can scale (multiply by a scalar) rv



Application: Parametric (& Symmetric) Eqn of a Line
(We draw for $n=2$)

To describe a line ℓ we need a vector in its direction $v \neq 0$ and a point P on the line



t is a parameter for the line

So $\ell = \{ P + tv \mid t \in \mathbb{R} \}$

Ex Find the line in $\mathbb{R}^3$ in direction $v = (1,2,3)$ through the point $P = (2,4,6)$

Ans: $\ell = \{ (2,4,6) + t(1,2,3) \mid t \in \mathbb{R} \}$
$= \{ (2,4,6) + (t, 2t, 3t) \mid t \in \mathbb{R} \}$

$\ell = \{ (\underset{x_1}{2+t}, \underbrace{4+2t}_{x_2}, \underbrace{6+3t}_{x_3}) \mid t \in \mathbb{R} \}$ parametric eqn of line

ie ℓ consists of the set of points $(x_1, x_2, x_3) \in \mathbb{R}^3$ where

$x_1 = 2+t \rightarrow t = x_1 - 2$

$x_2 = 4+2t \rightarrow t = (x_2-4)/2$

$x_3 = 6+3t \rightarrow t = (x_3-6)/3$

$$\Rightarrow (t =) \quad x_1 - 2 = \frac{x_2-4}{2} = \frac{x_3-6}{3}$$

(Symmetric) Equation(s) of Line ℓ

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So in general if $P = (P_1, P_2, P_3)$ & $v = (v_1, v_2, v_3)$;

The eqn is: $\boxed{\frac{x_1 - P_1}{v_1} = \frac{x_2 - P_2}{v_2} = \frac{x_3 - P_3}{v_3}}$ $(= t)$

Question: (to be answered later by solving simultaneous linear equations)

Let $\ell_1 : \frac{x_1-3}{4} = \frac{x_2-1}{2} = \frac{x_3+1}{3}$ $(=t)$

& $\ell_2 : x_1 - 1 = \frac{x_2-2}{4} = \frac{x_3-1}{2}$ $(=s)$

Do ℓ_1 & ℓ_2 intersect?

And if so, find the intersection?

In parametric form we have

ℓ_1: $P = (3, 1, -1)$, $v = (4, 2, 3)$ from box above

So $\ell_1 : P + tv = (3+4t, 1+2t, -1+3t)$, $t \in \mathbb{R}$

& ℓ_2: $x_1 - 1 = s \Rightarrow x_1 = s+1$

$\frac{x_2-2}{4} = s \Rightarrow x_2 = 4s+2$

$\Rightarrow x_3 = 2s+1$

So $\ell_2: (1+s, 2+4s, 1+2s), s \in \mathbb{R}$

So where ℓ_1 & ℓ_2 meet their x_1, x_2 & x_3 coordinates must be the same

ie
$$\left.\begin{aligned} 3+4t &= s+1 \\ 1+2t &= 4s+2 \\ -1+3t &= 1+2s \end{aligned}\right\}$$

or
$$\left.\begin{aligned} 4t-s &= -2 \\ 2t-4s &= 1 \\ 3t-2s &= 2 \end{aligned}\right\}$$ system of linear equations. Solve for s & t. (if possible)

We don't expect any solutions as:
Geometrically in general 2 lines won't meet in $\mathbb{R}^3$
& too many eqns ie 3 eqns in 2 unknowns

The Dot Product. (Encodes length and angle between vectors)

def^n: Let $u = (u_1, u_2, u_3, \dots, u_n)$ & $v = (v_1, v_2, \dots, v_n)$

Then the dot or scalar product of u & v is the real number denoted by $u \bullet v$ and defined by

$$u \bullet v = u_1 v_1 + u_2 v_2 + u_3 v_3 + \dots + u_n v_n$$
$$= \sum_{i=1}^{n} u_i v_i$$

Ex $u = (1,2,1)$ & $v = (-1,2,2)$

$$u \bullet v = 1(-1) + 2(2) + 1(2) = 5$$

(and note that $v \cdot u = 5$)

Ex: Let $u = (2,3)$ & $v = (-3,2)$

then $u \cdot v = 2(-3) + 3(2) = 0$

and $u \cdot u = 2(2) + 3(3)$
$= 2^2 + 3^2$ = the distance squared from $(0,0)$ to $u = (2,3)$

ie the length $\|u\|$ of the vector u squared.

in general, by Pythagoras we have if $u = (u_1, u_2) \in \mathbb{R}^2$ then

$\|u\|^2 = u_1 u_1 + u_2 u_2 = u_1^2 + u_2^2$

$\Rightarrow \|u\| = \sqrt{u_1^2 + u_2^2}$