

Programming Paradigms

CT331 Week 7 Lecture 2

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Example recursive function: Factorial

The factorial of a non-negative integer n , written $n!$, is the product of all integers less than or equal to n . The factorial of 0 is 1

Factorial

1. What is the base case? 0
2. What should the answer be when we are at the base case?
3. How do you reduce to get to this base case ?
4. What other work needs to be done for each function call?
5. (e.g., creating a new list, etc.)?
6. How can these steps be put together?

Factorial

1. What is the base case? **0 or 1?**
2. What should the answer be when we are at the base case? **The factorial of 0 is 1**
3. How do you reduce to get to this base case ?
4. What other work needs to be done for each function call?
5. (e.g., creating a new list, etc.)?
6. How can these steps be put together?

Factorial

(fact 0) => 1

(fact 1) => 1 * (fact 0)

(fact 2) => 2 * 1 * (fact 0)

Factorial

(fact 0) => 1

(fact 1) => 1 * (fact 0)

(fact 2) => 2 * 1 * (fact 0)



This is the same as (fact 1)

Factorial

(fact 0) => 1

(fact 1) => 1

(fact 2) => 2 * 1

(fact 3) => 3 * 2 * 1



This is the same as (fact 2)

(fact 4) => 4 * 3 * 2 * 1



This is the same as (fact 3)

Factorial

$$(\text{fact } 0) \Rightarrow 1$$

$$(\text{fact } 1) \Rightarrow 1$$

$$(\text{fact } 2) \Rightarrow 2 * 1$$

$$(\text{fact } 3) \Rightarrow 3 * 2 * 1$$



This is the same as (fact 2)

$$(\text{fact } 4) \Rightarrow 4 * 3 * 2 * 1$$



This is the same as (fact 3)

$$(\text{fact } 3) \Rightarrow 3 * (\text{fact } 2)$$

$$(\text{fact } 4) \Rightarrow 4 * (\text{fact } 3)$$

Factorial

1. What is the base case? **0**
2. What should the answer be when we are at the base case? **The factorial of 0 is 1**
3. How do you reduce to get to this base case ? **$n * (n-1) * (n-2) * (n-3) \dots$**
4. What other work needs to be done for each function call?
5. (e.g., creating a new list, etc.)?
6. How can these steps be put together?

Factorial

1. What is the base case? **0**
2. What should the answer be when we are at the base case? **The factorial of 0 is 1**
3. How do you reduce to get to this base case ? **$n * (n-1) * (n-2) * (n-3) \dots$**
4. What other work needs to be done for each function call?
- 5. Subtract 1 from n, multiply result**
6. How can these steps be put together? Simpler example first

Example recursive function: sum

Write a scheme function that takes a number n and returns the sum of the numbers up to n .

Example (sum 4) will return 10

Recursive sum function

```
(define (sum num)
  (if (> num 1)
      (+ num (sum (- num 1)))
      1))
```

Recursive factorial function

```
(define (factorial num)
  (if (> num 1)
      (* num (factorial (- num 1)))
      1))
```

Very similar to sum

Example recursive function: all atom?

Write a recursive function `all_atoms?` which checks if all elements in a list are symbols returning `#t` or `#f`, e.g.,

```
> (all_atoms? '(x y z))
#t
> (all_atoms? '())
#t
> (all_atoms? '(a (b) c))
#f
```

Hint:
you will need to make use of the built-in predicate `atom?`

Other functions/predicates needed: `define` `cond` `null?` `car` `cdr` `not`

Example recursive function: all atom?

Base cases:

- (null? lst) stop and output #t
- (car lst) not an atom, stop and output #f
- One element in list and it is a atom, stop and output #t

Reduce:

- if (car lst) is a atom, continue and check (cdr lst).

Work through this problem – we can look at it in tutorial this week

Example recursive function: sequence

Write a (recursive) function `sequence` which is passed a number `num` and creates a list of numbers from 1 to `num`.

Note: Can write this function only using functions/predicates we have seen:

```
define
cond
append
list
=
'()
```


Example recursive function: sequence

Base case: If num is 0?

Reduce: - 1 from num

Work: append num to end of new list

Write the code for this function – again we can look at it in the tutorial

Is it the most efficient solution?

Example recursive function: max

The built-in function `max` returns the maximum of a set of numbers, e.g.,
`(max 3 5 4 19 7)` returns 19.

Write your own version of a function to find the maximum of a list of numbers,
e.g., `(maxlst '(3 5 4 19 7))` returns 19

Example recursive function: max

Base case:

one element in list - it's the maximum

Reduce:

Keep checking pairs of items in list, removing the smaller one, until we have reduced to a list with one element

Example recursive function: max

Base case:

One element in list?

```
(null? (cdr lst))
```

Return that element

```
(car lst)
```

Example recursive function: max

Recursive case: Checking pairs of elements

If first element bigger than second one

```
(> (car lst) (cadr lst))
```

remove (cadr lst) from list which will be sent to recursive call:

```
(cons (car lst) (cddr lst))
```

If second element is bigger, ignore (car lst) for recursive call.

```
(cdr lst)
```

Example recursive function: max

```
(define (maxlst lst)
  (cond [ (null? lst) (display '(empty list)) ]
        [ (null? (cdr lst)) (car lst) ]
        [ (> (car lst) (cadr lst))
          (maxlst (cons (car lst) (cddr lst))) ]
        [ else (maxlst (cdr lst)) ] ))
```