

## CS4423: Assignment 2: Part 1 - Tutorial Sheet

**Tutorial sheet for working on in classes**

These exercises are similar Q5–Q9 on Homework Assignment 2: Part 1. They can be covered in class by the tutor. For the actual assignment, you need to answer the questions at [www.niallmadden.ie/2425-CS4423/CS4423-HW2-1.pdf](http://www.niallmadden.ie/2425-CS4423/CS4423-HW2-1.pdf)

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Some background: *The Network Laplacian*

**Graph Laplacian.** There are many ways to represent a network as a matrix, such as the adjacency matrix. Another is the *Laplacian*,  $L = (l_{ij})$ . For a network  $G = (X, E)$  of order  $n$  with nodes labelled  $1, 2, \dots, n$ ,  $L$  is the square  $n \times n$  matrix with entries

$$l_{ij} = \begin{cases} \deg(i) & i = j \\ -1 & \{i, j\} \in X \\ 0 & \text{otherwise} \end{cases}$$

For example, if  $G = K_3$ , then

$$L = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

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(a) Let  $G_1$  be the tree on the nodes  $\{0, 1, 2, \dots, 9\}$  with Laplacian matrix

$$L = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

Sketch  $G_1$ .

(b) Give the Prüfer code for  $G_1$ .

(c) Sketch the tree,  $T_2$ , on the nodes  $\{0, 1, \dots, 9\}$  that has as its Prüfer code  $[0, 5, 0, 6, 0, 7, 0, 8]$ .

(d) Give the order in which the nodes of  $T_2$  would be visited if it is traversed by **Depth First Search**.

(e) Give the order in which the nodes of  $T_2$  would be visited if it is traversed by *Breadth First Search*.